Graphical and Simplex solutions to problem 2.3: Allocating Crops with land and water constraints

Problem from CEE 5410 Course Notes (Bishop et. al, 1999)

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# Introduction

An aqueduct supplying industrial interests has an excess capacity in June, July, and August of 14,000, 18,000, and 6,000 acre-ft of water, respectively. The excess water is valuable for developing irrigated farming on nearby land. The land developer stated not more than 10,000 acres of land will be allocated to crops and irrigated using the water. Each crop requires varying amounts of water depending on the month as shown in Table 3 (Bishop, Hughes and McKee 1999, 36). This report discusses the land allocations for hay and grain crops that maximize the financial returns from the land use. The graphical solution and the simplex algorithm are summarized, followed by calculation results and a summary of key insights from the model. Appendix A contains selected tables and figures, and Appendix B contains full calculations.

# Methods

Both a graphical solution and the Simplex algorithm were used to arrive at the optimum land areas for each crop. The following steps outline the solution process for each method:

* Define the decision variables. In this case, the decision variables include the area of hay crops (Ah) and area of grain crops (Ag) the developer will plant.
* Define the objective function describing the financial return from the allocated land resources among the decision variables as shown in Figure 4.
* Define the constraints on the problem that govern the feasible solution space, including slack variables for simplex methods, as shown in Figure 3 and Figure 5.
* For graphical methods, plot the constraint inequalities and the objective function slope for the maximum return value through a basic feasible solution (Rosenberg 2020).
* For simplex methods, minimize the negative objective function (Figure 4) by using the simplex algorithm. The simplex algorithm uses Gauss-Jordan elimination to iteratively assign variables to a 0 value until the identity matrix in the decision variables indicates the optimal solution, as shown in Table 3 (Bishop, Hughes and McKee 1999, 39-52).

# Results

The graphical and simplex solutions indicate the maximum return of $1.16 million will occur when 2,000 acres of hay and 8,000 acres of grain grow on the field. Table 1 contains insights about leftover resources and additional profit values of binding constraints.

Table 1. Slack and shadow values of constraint variables found from simplex solutions

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Slack Variable** | **Status** | **Slack Value** | **Shadow Value** | **Interpretation** |
| Land constraint | Binding | 0 | 80 | Profit will increase $80 per add. acre of land available |
| June Water | Non-binding | 2000 | 0 | 2000 ac-ft of excess water will remain in June, no profit increase |
| July Water | Binding | 0 | 20 | Profit increases $20 per add. ac-ft of excess water from aqueduct |
| August Water | Non-binding | 4000 | 0 | 4000 ac-ft of excess water will remain in August, no profit increase |

# Conclusion

The land developer should plant 2000 acres of hay and 8000 acres of grain crops. The only constraints to consider in this scenario are the land constraint (10,000 acres of total crop area) and the available water in July (18,000 acre-ft). Each additional acre of land and additional acre-ft of water in July would increase total returns by $80 and $20, respectively.

# References

Bishop, A Bruce; Hughes, Trevor; and McKee, Mac, "Water Resources Systems Analysis - Course Notes" (1999). *All ECSTATIC Materials.* Paper 76.  
https://digitalcommons.usu.edu/ecstatic\_all/76

Rosenberg, David. 2020. "Linear Program Formulation and Graphical Solution (Course lecture)." *CEE 5410/6410: Water Resource Systems Analysis.* Logan: Utah State University, September 4.

# Appendix A: Selected Tables and Figures

This section contains selected tables and figures to aid the reader in understanding the methodology in the calculations process. For a full calculation sheet, please refer to Appendix B.

Table 2. Decision variable descriptions

|  |  |  |
| --- | --- | --- |
| **Decision variables** | | |
| Symbol | Units | Description |
| Ah | acre | Area of land allocated to hay crops |
| Ag | acre | Area of land allocated to grain crops |

## Graphical solution

Table 3. Monthly water requirements (acre-ft)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Crop | June | July | August | Return, $/acre |
| Hay | 2 | 1 | 1 | 100 |
| Grain | 1 | 2 | 0 | 120 |

Figure 1. Graphical representation of convex set and optimal solution

Figure 4. Objective functions for graphical and simplex method

***Constraint Equations - Graphical***

Figure 3: Graphical constraint inequalities

**For optimal solution, objective function lies along the line**

Figure 2. Objective function (optimum solution)

## Simplex Solution

***Constraint Equations - Simplex***

Figure 5. Simplex constraint equations with new slack variables

Table 4. Tableaus in simplex solution

|  |
| --- |
| **Tableau 1** |
| **Item** | **Ah** | **Ag** | **S3** | **S4** | **S5** | **S6** | **b** | **-Z** | **Most - Obj Func. Coeff** | **Pivot Variable** | **Binding: bi/aij** |
| Value | **0** | **0** | **10000** | **14000** | **18000** | **6000** |  |  |  |  |  |
| Obj. Z Coeff | -100 | -120 | 0 | 0 | 0 | 0 |  | $0.00 | -120 | 2 |  |
| C3 | 1 | 1 | 1 | 0 | 0 | 0 | 10000 |  | Continue |  | 10000 |
| C4 | 2 | 1 | 0 | 1 | 0 | 0 | 14000 |  |  |  | 14000 |
| C5 | 1 | 2 | 0 | 0 | 1 | 0 | 18000 |  |  |  | 9000 |
| C6 | 1 | 0 | 0 | 0 | 0 | 1 | 6000 |  |  |  | #DIV/0! |
| **Tableau 2** |
| **Item** | **Ah** | **Ag** | **S3** | **S4** | **S5** | **S6** | **b** | **-Z** | **Most - Obj Func. Coeff** | **Pivot Variable** | **Binding: bi/aij** |
| Value | 0 | 9000 | 1000 | 14000 | 0 | 6000 |  |  |  |  |  |
| Obj. Z Coeff | -40 | 0 | 0 | 0 | 60 | 0 |  | -$1,080,000.00 | -40 | 1 |  |
| C3 | 0.5 | 0 | 1 | 0 | -0.5 | 0 | 1000 |  | Continue |  | 2000 |
| C4 | 1.5 | 0 | 0 | 1 | -0.5 | 0 | 5000 |  |  |  | 3333.3333 |
| C5 | 0.5 | 1 | 0 | 0 | 0.5 | 0 | 9000 |  |  |  | 18000 |
| C6 | 1 | 0 | 0 | 0 | 0 | 1 | 6000 |  |  |  | 6000 |
| **Tableau 3** |
| **Item** | **Ah** | **Ag** | **S3** | **S4** | **S5** | **S6** | **b** | **-Z** | **Most - Obj Func. Coeff** | **Pivot Variable** | **Binding: bi/aij** |
| Value | 2000 | 8000 | 0 | 2000 | 0 | 4000 |  |  |  |  |  |
| Obj. Z Coeff | 0 | 0 | 80 | 0 | 20 | 0 |  | -$1,160,000.00 | 0 | 1 |  |
| C3 | 1 | 0 | 2 | 0 | -1 | 0 | 2000 |  | STOP |  | #DIV/0! |
| C4 | 0 | 0 | -3 | 1 | 1 | 0 | 2000 |  |  |  | #DIV/0! |
| C5 | 0 | 1 | -1 | 0 | 1 | 0 | 8000 |  |  |  | 8000 |
| C6 | 0 | 0 | -2 | 0 | 1 | 1 | 4000 |  |  |  | #DIV/0! |

# Appendix B: Full Calculation Sheet



A screenshot of a social media post

Description automatically generated

A screenshot of text

Description automatically generated

A screenshot of a social media post

Description automatically generated

A picture containing clock

Description automatically generated

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **Category**  **(Max. Score)** | **No Evidence** | **Doesn’t Meet Standard** | **Nearly Meets Standard** | **Meets Standard** | **Exceeds Standard** | **Self- Score** | **Instructor Score** |
| **Title**  **(1)** | Absent  0 | Evidence of two or less  0 | Evidence of three  0 | Evidence of four  1 | Title – can assess main point from title alone; Name, Instructors’ Names, Course, Date, Neatly finished 1 | 1 |  |
| **Introduction**  **(3)** | Absent, no evidence  0 | There is no clear introduction or main topic.  1 | Introduction states the main topic but either:   1. Does not give a full overview, Or: 2. Too detailed, leading to annoying repetition later. 2 | The introduction states the main topic and previews the structure of the report.  2 | The introduction states the main topic and previews the structure of the report. Good overview of the problem and solution approach. Gives enough detail to motivate the reader to continue reading.  3 | 3 |  |
| **Organization and structural development of the idea: procedure, results, conclusions**  **(10)** | No content provided.  0 | Paragraphs fail to develop the main idea. No section headers or guide to help the reader understand how material is organized.  1 – 5 | Organization of ideas not fully developed. Paragraphs lack supporting detail sentences. No transitions and/or ineffective section headers.  6 - 7 | Paragraph development present but not perfected. Each paragraph has sufficient supporting detail sentences. Few transitions.  8 | Writer demonstrates logic and sequencing of ideas through well-developed section headers, paragraphs, and transitions. The first sentence of each paragraph is the summary sentence.  9 - 10 | 9 |  |
| **Technical Correctness**  **(70)** | Questions not addressed.  3 – 42% | The writer has no clue what they are talking about.  45 – 58% | Sketchy: left out required design points. Did not work on this as much as you should have, and it shows. Many important answers are incorrect.  61 – 79% | Discussion lacks adequate detail, but all the necessary points are covered and nearly all answers are correct.  82 – 88% | Provides what was explicitly asked for. The function of each piece is demonstrated to the reader in adequate, but not overwhelming, detail. Answers are correct and reasonable.  91 – 100% |  |  |
| 1. Problem Formulation (20) | | | | |  |  |
| 1. Decision Variables (5) | | | | | 5 |  |
| 1. Objective Function (5) | | | | | 5 |  |
| 1. Constraints Equations (10) | | | | | 10 |  |
| 1. Graphical Solution (25) | | | | |  |  |
| 1. Plot Constraints, Identify Convex Set (10) | | | | | 10 |  |
| 1. Plot Objective Function Slope (10) | | | | | 10 |  |
| 1. Determine Optimal Solution Extreme Point (5) | | | | | 5 |  |
| 1. Simplex Solution (25) | | | | |  |  |
| 1. Initial basis (5) | | | | | 5 |  |
| 1. Tableaus (15) | | | | | 15 |  |
| 1. Interpretation of solution from final tableau (5) | | | | | 5 |  |
| **Category**  **(Max. Score)** | **No Evidence** | **Doesn’t Meet Standard** | **Nearly Meets Standard** | **Meets Standard** | **Exceeds Standard** | **Self- Score** | **Instructor Score** |
| **Word Usage and Format**  **(10)** | Not applicable | Numerous and distracting errors in punctuation, capitalization, spelling, sentence structure, word usage, significant figures, tables, and figures. Data vomited onto page(s). Unacceptable / unprofessional at the graduate level. 1 – 5 | Misspelled words, poor English grammar and word choice. Main body of report is either longer or significantly less than one page. Figures are too small and/or under-labeled, although they are usually of acceptable quality and focus. Tables incoherent or not cohesive. Bad font sizes. Too much or too little data in appendices. Could be improved by being more meticulous.  6 - 7 | Almost no errors in punctuation, capitalization, spelling, sentence structure, word usage, significant figures, and presentation of figures, tables, and appendices. Main body of report is one page or less  8 | Punctuation, capitalization, spelling, sentence structure, word usage, and significant figures all correct. Main body of report is one page or less. Clear, consistent fonts. Good word processing skills. Figures have adequate contrast. Informative figure and table titles and legends. Figures have appropriate axis tick spacing, labels, units, and legends. Table columns cohesive, labeled, and specify units. Document is stapled. Appendices, if provided, are separated by topic, and each have a title, discussion, and proper formatting and display of information 9 - 10 | 9 |  |
| **Conclusion**  **(4)** | Absent  0 | Incomplete and/or not focused. 1 | The conclusion does not adequately restate the main results. 2 | The conclusion restates the main results. 3 | The conclusion restates the main results, and is an effective summary. 4 | 4 |  |
| **References**  **(0)** | Absent  0 | Numerous errors, off-the-wall sources used. 0 | Some errors in citing format; more sources should be cited.  1 | Prior work cited with few errors.  2 | All prior work and data sources are cited in the correct format with no errors.  2 | 0 |  |
| **TOTAL** (98) |  | | | | | 96 |  |